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Variational Approach to Trajectory Optimization w. r. t. Energy Recuperation for Stacker Cranes

Variationsmethoden zur Trajektorienoptimierung bezüglich Energierekuperation bei Regalbediengeräten

Rico Zöllner
Ella Jannasch
Thorsten Schmidt

Institute of Material Handling and Industrial Engineering,
TU Dresden, GERMANY

For optimal motion planning of stacker cranes, a time-continuous model based on a variational approach is presented. The objective is to maximize energy recuperation between the two drives of a stacker crane. In addition to a comparison with a time-discrete approach, the energy saving potential as well as technically adverse trajectories are identified for a concrete example.

[Keywords: Trajectory Optimization, Energy Recuperation, Non-smooth Analysis, Warehouse Logistics, Stacker Cranes]

Zur optimalen Bahnplanung von Regalbediengeräten wird ein zeit-kontinuierliches Modell auf Basis eines Variationsansatzes entwickelt. Das Ziel besteht in der Maximierung der Energierekuperation zwischen den beiden Antrieben des Gerätes. Neben einem Vergleich zu einem zeit-diskreten Ansatz werden für ein Beispiel Energiesparpotentiale, aber auch technisch nachteilige Trajektorien aufgezeigt.

[Schlüsselwörter: Trajektorienoptimierung, Energierekuperation, Nichtglatte Analysis, Lagerlogistik, Regalbediengeräte]

1 Introduction

Energy saving plays an important role, and stacker cranes make up a remarkable share of the energy consumption of a warehouse due to their dimensions (see Figure 1). Some of the possibilities for saving energy have other disadvantages. Due to the sizes of existing warehouses and the loads to be transported, it is not possible to downsize the vehicles. The installation of energy storage systems generates additional costs. In addition, loss-free storage is impossible. Reducing the speed of movement would reduce the warehouse's throughput. The aim of the present paper is to optimize the *way* of moving (trajectories) so that the recuperation between the two drives (running gear, lifting gear) of a stacker crane is used optimally, but without reducing the throughput.

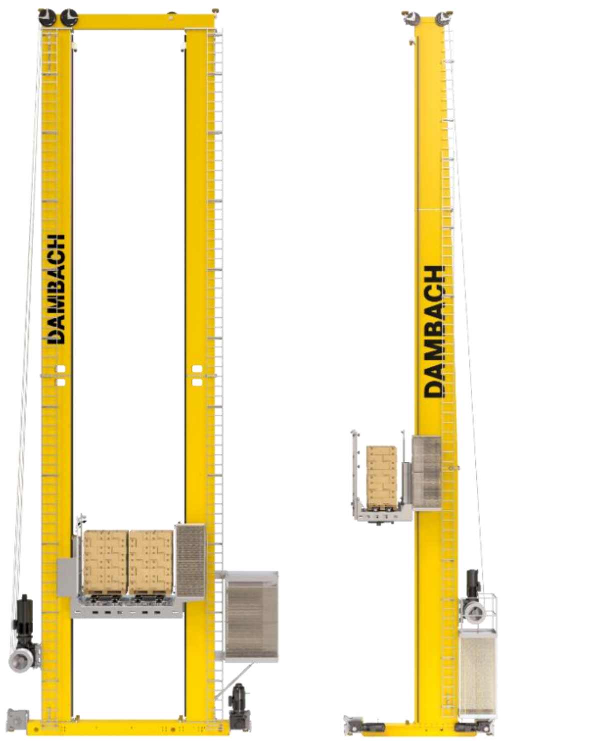

Figure 1 Example of a stacker crane (front view left, side view right). The total mass of up to 40 tons in combination with typical heights of 30 meters cause power peaks of several 100 kW. Regarding the front view, the two drives are colored in black, and the lifting gear is located at the left side and the running gear at the bottom (credit: DAMBACH GmbH & Co. KG).

This is done in the following way: for each travel, one of the drives is time-critical, namely that one that takes the longer to move at maximum speed (maximum acceleration and jerk). This drive (called the slow) should continue to run in a time-minimal manner, whereas the velocity profile of the other drive (called the fast) is adjusted so that as much power as possible is used directly on the vehicle, so that neither an unnecessary surplus is created nor unnecessary power is drawn from the power grid. Therefore, for each time point t the sum over all $|P_{\text{slow}}(t) + P_{\text{fast}}(t)|$ is to be minimized.

The power flow within the two power trains of a stacker crane is described by intricated non-linear functions to model their technical features (mechanics and electrics). Thus, time-discrete methods for trajectory optimization tend to numerical instabilities, at the same time requiring advanced computational power (especially if an a priori equi-distant time grid is used). We propose an efficient and real-time capable method based on non-smooth variational techniques (Clarke gradients [1]) for deriving optimal trajectories and their characteristic features enabling the computation of a large number of trajectories serving as a potential data base for subsequent analyses and optimizations.

Taking a glimpse at literature, [2] provides a review on optimization. Motion planning for cranes is the topic of [3, 4], for instance. To get insights into direct methods (i. e. discrete optimization) we recommend for example [5–8], and [3, 9–11] for indirect and variational methods. A general review on related topics of warehouse logistics can be found in [12].

Our paper is organized as follows: Section 2.1 describes the problem set-up in detail before the standard time-discrete model is sketched in Section 2.2. The mathematical results of our variational time-continuous model are collected in Section 2.3 (see [13] for details). Section 3 is dedicated to the results of a numerical case study. As a first step, both models are compared w. r. t. computational time and quality of the results. The second step is a systematic scan over the plane of possible movements to show the potential amount of energy saving and some special cases of trajectories. Section 4 summarizes the paper.

2 Methods

2.1 Optimization Task

The vehicle has to move from a start point A to an end point B (see Figure 2). Thus, the horizontal drive has to overcome the distance s_x obeying bounds of the velocity v_x, the acceleration a_x and the jerk j_x. Analogously, the vertical drive has to overcome the distance s_y obeying its individual bounds. If they would move time minimally, both single movements would yield time durations T_x and T_y, respectively. To not reduce the throughput, the entire time horizon T is defined as $T = \max\{T_x, T_y\}$ which forces one of the drives (the slow) to move time minimally. Its velocity profile and its power profile are fixed by this requirement. The trajectory of the other drive (the fast) is optimized such that the energy recuperation between the two drives is maximally used. To evaluate the energy and the power for each drive, let there be a function $P = P(v, a)$ called power flow model (see [14] for details). Due to technical reasons, the velocity profiles v_x and v_y have to be monotone (as functions of the time t). In addition, the rectangle spanned by the points A and B must not be left.

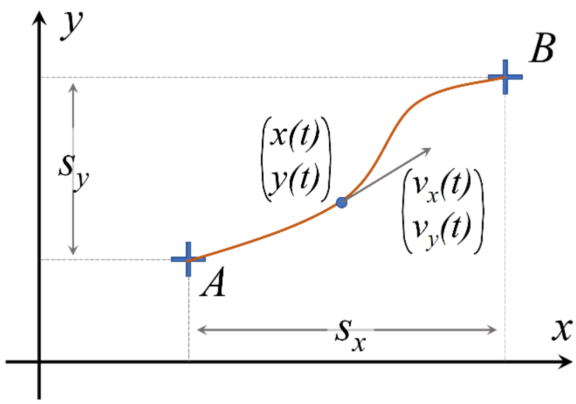

Figure 2 Geometric aspects of the trajectory optimization, where the vehicle starts from point A and moves to point B, in such a way that one of the drives – exactly that one which needs more time to overcome the prescribed distance – behaves time minimally and the other one makes optimal use of the recuperated energy.

The following two sections present precise mathematical formulations of the optimization task for that drive which does not move time minimally (the fast drive). The symbols v, a and j refer to the fast drive; coordinate indices are suppressed.

2.2 Direct Discrete Approach

Direct optimization methods perform an a priori time discretization. Consider an equi-distant decomposition $(t_k)_{k=0}^{N}$ of the time interval $[0, T]$ with $t_{k+1} - t_k = \Delta t$, $t_0 = 0$ and $t_N = T$. The jerk j is assumed to be constant over the interval $[t_k, t_{k+1}]$ (denoted by j_k, i. e. $j_k := j(t_k)$ and s, v, a analogously) and taken as the target variable of the optimization. Defining the associated state quantities (acceleration, velocity, distance) by

$$a_{k+1} = a_k + j_k \cdot \Delta t \tag{1}$$

$$v_{k+1} = v_k + a_k \cdot \Delta t + \tfrac{1}{2} j_k \cdot \Delta t^2 \tag{2}$$

$$s_{k+1} = s_k + v_k \cdot \Delta t + \tfrac{1}{2} a_k \cdot \Delta t^2 + \tfrac{1}{6} j_k \cdot \Delta t^3 \tag{3}$$

leads to the following set of boundary and side conditions:

$$j_0 = j_N = a_0 = a_N = v_0 = v_N = 0 \tag{4}$$

$$s_N - s_0 = s_{x/y} \tag{5}$$

$$(s_{k+1} - s_k) \cdot \operatorname{sgn} s_{x/y} \geq 0 \tag{6}$$

$$|j_k| \leq j_{\max} \tag{7}$$

$$|a_k| \leq a_{\max} \tag{8}$$

$$|v_k| \leq v_{\max} \tag{9}$$

Note that the index x/y in Equations (5-6) depends on which drive is the fast one. The signum in Equation (6) distinguishes up/down or left/right travels. Equation (6) both ensures the monotony and the movement within the rectangle between point A and point B. The objective function reads as

$$E = \sum_{k=0}^{N} |P_{\text{slow}}(t_k) + P_{\text{fast}}(v_k, a_k)| \rightarrow \min \tag{10}$$

Where the absolute value measures the recuperation in such a way that the total power flow in both directions should be minimized.
The direct approach leads to a finite (but high) dimensional optimization problem to which an appropriate solver has to be applied (see Section 3.1 for details).

2.3 Indirect Variational Approach

Indirect methods consider time continuous quantities and develop necessary optimality conditions (before a subsequent, not necessarily equi-distant discretization is applied). Regarding the time interval $[0, T]$, the boundary and side conditions now read as

$$v(0) = v(T) = \dot{v}(0) = \dot{v}(T) = 0 \tag{11}$$

$$G(v) := \int_0^T v(t)\text{d}t - s_{x/y} = 0 \tag{12}$$

$$g_v(v) := \max_{t\in[0,T]} \left| v(t) + \tfrac{1}{2}\text{sgn}(s_{x/y}) v_{\max} \right| - \tfrac{1}{2} v_{\max} \leq 0 \tag{13}$$

$$g_a(v) := \max_{t\in[0,T]} |\dot{v}(t)| - a_{\max} \leq 0 \tag{14}$$

$$g_j(v) := \max_{t\in[0,T]} |\ddot{v}(t)| - j_{\max} \leq 0 \tag{15}$$

Where the dot denotes the time derivative and $\dot{v} = a$ and $\ddot{v} = j$. The objective function is given by the integral

$$E := \int_0^T |P_{\text{slow}}(t) + P_{\text{fast}}(v, \dot{v})| \text{d}t \rightarrow \min \tag{16}$$

Finding the optimality conditions means applying sophisticated analytic tools (known as Euler-Lagrange formalism and Pontryagin's principle), see [13] for detailed calculations and further references.
The investigations done in [13] reveal that the optimal trajectory consists of a series of time intervals $[\tau_n, \tau_{k+1}]$ covering $[0, T]$ where either one of the side conditions (Equations (13-15)) is active or the function $v(t)$ is a solution of the Euler-Lagrange equation

$$\frac{\partial P_{\text{fast}}}{\partial v} - \frac{\text{d}}{\text{d}t}\frac{\partial P_{\text{fast}}}{\partial \dot{v}} + \lambda_G = 0 \tag{17}$$

with a Lagrange multiplier λ_G (arising from Equation (12)) to be determined.
Finding the optimum is equivalent to the determination of the time grid $(\tau_k)_{k=0}^{M}$ and the respective solution of Equations (13-15) or (17), see Section 3.1 for details.

3 Results

We consider a concrete example of a stacker crane. Therefore, the power flow model as a functional relation between the velocity v, the acceleration a, the load mass m and the power P is fixed (see [13–15] for remarks, Figure 2 provides an impression). In addition, the parameters referring to technical boundaries are listed in Table 1.

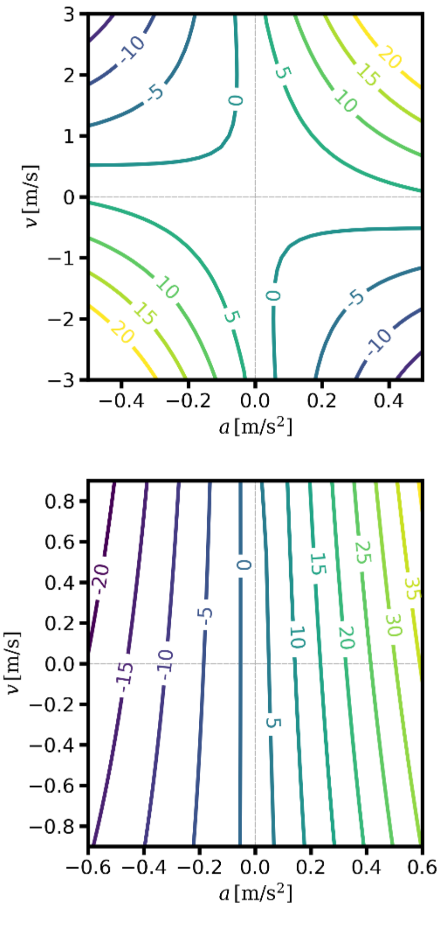

Figure 3 Contour plot of the power $P(v, a)$ in kW as a function of the velocity v and the acceleration a for a load mass of 1000 kg (upper panel: running gear, lower panel: lifting gear).

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parameter	running gear	lifting gear
$v_{\max}$	$3.0\ \mathrm{m/s}$	$0.9\ \mathrm{m/s}$
$a_{\max}$	$0.5\ \mathrm{m/s^2}$	$0.6\ \mathrm{m/s^2}$
$j_{\max}$	$1.0\ \mathrm{m/s^3}$	$0.6\ \mathrm{m/s^3}$

Table 1 Kinematic parameters $v_{\max}$, $a_{\max}$, $j_{\max}$ defining the velocity, acceleration and jerk bounds, respectively, and entering the optimization problem in Equations (7-9, 13-15).

3.1 Direct vs. Indirect Approach

This section contains a comparison of the time-discrete model of Section 2.2 and the variational time-continuous model of Section 2.3. The discrete model was implemented in Matlab (with the solver FMINCON) with a time step $\Delta t = 0.08$ s. The latter model was implemented in Python (with the solver ODEINT to tackle differential equations as core part). Let us consider the optimal trajectory connecting the points A and B with $s_x = 15$ m and $s_y = 18$ m. Figure 4 shows the shape of the optimal trajectory.

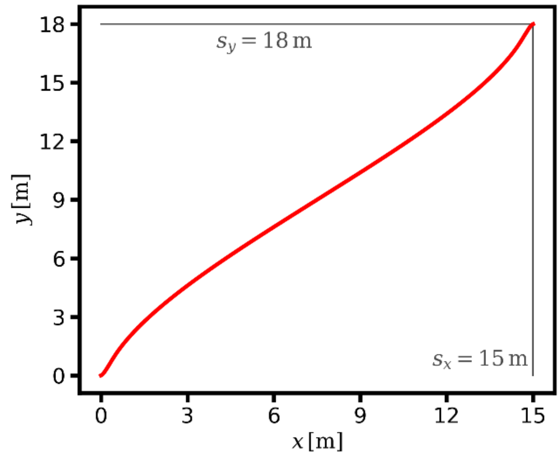

Figure 4 Shape of the optimal trajectory connecting two points with $s_x = 15$ m and $s_y = 18$ m. The lifting gear moves time minimally whereas the running gear is optimized w. r. t. maximal energy recuperation. The picture is generated with the continuous model but note however that the differences between the continuous and the discrete model are not visible by just inspecting the trajectory as such (see subsequent Figures 5 and 6).

The profiles of the velocity, the acceleration and the jerk reveal significant differences between both methods (see Figure 5 for the discrete model, Figure 6 for the continuous model).

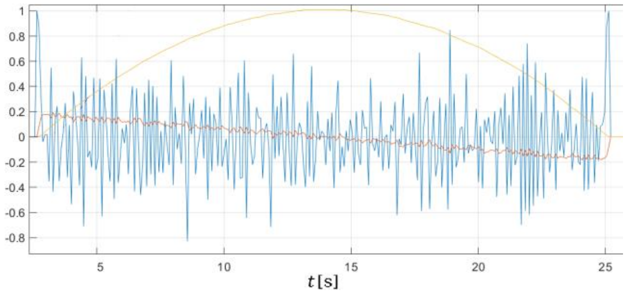

Figure 5 Profiles of the velocity v_x (yellow, m/s), the acceleration a_x (red, $\mathrm{m/s^2}$) and the jerk j_x (blue, $\mathrm{m/s^3}$) of the running gear for the trajectory displayed in Figure 4 as a result of the time-discrete model (Section 2.2). The oscillations result from an equi-distant discretization.

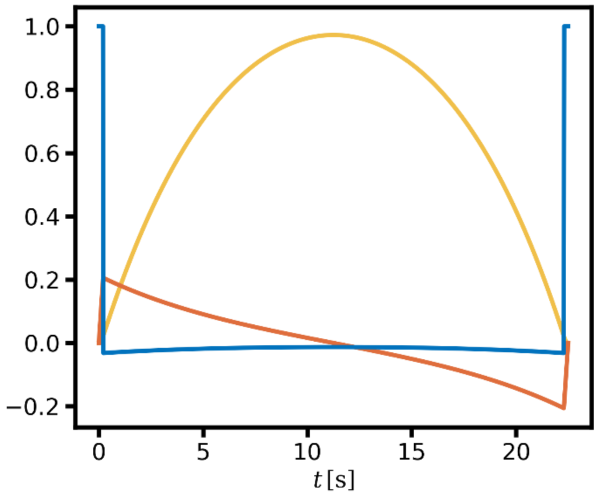

Figure 6 Analogous profiles to Figure 5 obtained with the time-continuous approach (Section 2.3). It can be seen that the movement actually consists of only three phases (M=3), where at the beginning and at the end of the travel the constraint (15) is active; in between the trajectory is a solution of the Euler-Lagrange equation (17). At the transition points, the jerk jumps and the acceleration and the velocity are continuous and differentiable (as a function of the time t), respectively.

The key points are as follows:

- The computational time needed for the discrete model is at least 100 times higher than for the continuous model. Since the discrete model deals with 283 time grid points ($N = 282$) whereas the continuous modes needs only 4 ($M = 3$).
- The obvious numerical instabilities might have several reasons:
    - Non-linearities are only catched approximately (especially side conditions are locally substituted by linear approximations).

- Non-smooth effects caused by constraints are approximated by smooth functions and classical optimizers exhibit an ill convergence if the smooth function lies close to the original one [16].
- The equi-distant time grid does not meet essential time points of the travel (see Figures 5 and 6).

From now on, we use the continuous approach to elaborate further features of optimal trajectories.

3.2 Systematics of Optimal Trajectories

Keeping the parameters of Tabel 1 as well as a load mass of 1000 kg fixed, the Figures 7 and 8 display the energy saving in % comparing the energy consumption of the optimized trajectories with the case were both drive move time-minimally (for up and down travels, respectively). Regarding the up travels of Figure 7, there are at most three phases during each movement (i. e. $M = 3$ and $M \ll N$). In addition, the energy saving rate is much higher if the lifting gear is adapted.

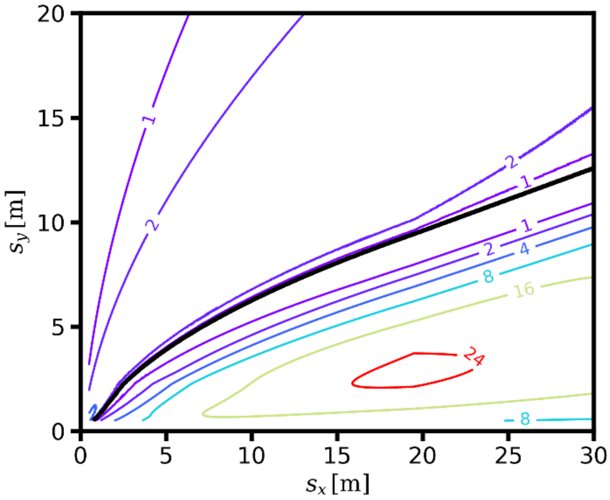

Figure 7 Contour plot of the energy saving in % comparing the optimized trajectories with trajectories where both drives move time-minimally for up movements. The black border curve indicates cases where both drives need the same time to move. Clearly, the saving is higher if the lifting gear is adapted (below the black curve).

Regarding the down travels (Figure 8), the optimal solution in most cases, where the lifting gear is adjusted, match the time-minimal solution, i. e. both drives should move time-minimally. In addition, there are cases (red shaded area) where the optimum of the running gear is to start and stop multiple times to dissipate the surplus of energy coming from the down movement of the lifting gear. Such trajectories are technically highly adverse (see Figure 9 for an example).

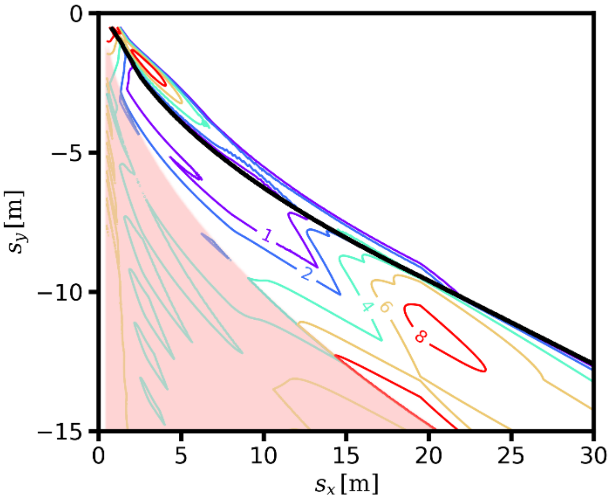

Figure 8 Contour plot of energy saving as in Figure 7 for down movements. In most of the cases above the black border curve there is no energy saving since the optimal trajectory and the time-minimal trajectory coincide. The red shaded area indicates cases where the optimal solution (for the running gear) consists of multiple starts and stops (see Figure 9 for an example). Such solutions should not be realized during the operation.

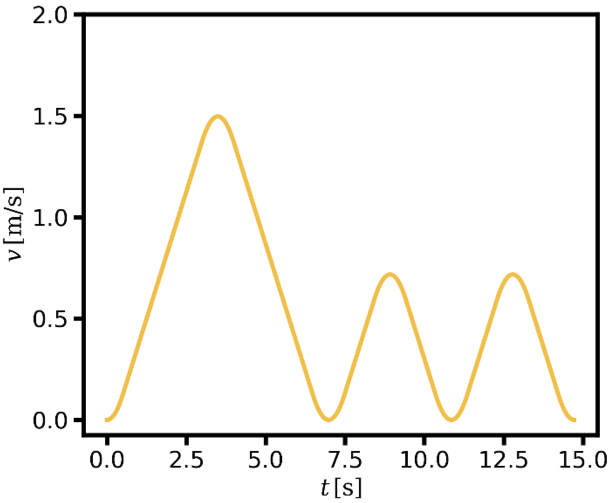

Figure 9 Velocity profile v_x (running gear) for the optimal trajectory with $s_x = 8\,m$ and s_y = -11 m. Since start-ups are rather expensive (in the sense of energy consumption) they occur to deal with the energy surplus provide by the lifting gear moving downwards. Note that the solution consists only of phases with active constraints (14, 15).

4 Conclusion

The present case study of energy recuperation for stacker cranes consists of two main parts: a methodical part with variational methods and a necessary optimality condition and a numerical part with a system of characteristic trajectory types. Regarding the method of optimization, an

a priori equi-distant time discretization seems as a charming easily applicable approach smoothly to implement. But however, direct methods make no use of special features of the given problem. This has to be compensated by computational power but still suffering from numerical instabilities due to non-linearities and a prejudicial choice of grid points. Contrarily, indirect methods require some efforts in the application of advanced mathematical tools. The benefit lies in a significant reduction of computational time by developing additional necessary conditions. Such conditions reveal further individual features of the problem and improve the deeper understanding of the problem.
Regarding the numerical results, there are several implications:

- The optimal trajectories consist of a rather small number of phases with a certain type of dynamics. This makes it more important for the optimization method to find the transition time points of those phases (non-equi-distant time grid is subject to the optimization).
- The type of dynamics is either determined by an active constraint or by the solution of an equation à la Euler-Lagrange.
- A particular type of trajectory was found which occurs in case of longer down travels. Solely maximizing the recuperation forces the trajectory to form oscillations (starting and stopping the running gear multiple times) which are both mechanically and electrically highly adverse.

All in all, the aim of generally maximizing the recuperation should be questioned. In the expectational cases, the energy optimal trajectory has to be substituted by a suitable alternative. Through elaborating the systematics of optimal trajectories by analytic tools time consuming computations become superfluous, and the operator in practice can rely on a simple scheme of estimating the energy saving and for selecting the appropriate way to move. The calculations contribute to a reliable cost-benefit estimation. For future work, the presented methods will be extended to solve related tasks efficiently.

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RICO ZÖLLNER is a doctoral researcher at the Chair of Logistics Engineering at Technical University Dresden. He holds a Dipl.-Math. from the same university. His research interests are applied variational methods for optimization and tribology in materials handling. His e-mail adress is rico.zoellner@tu-dresden.de.

ELLA JANNASCH is a student research assistant at the chair of Logistics Engineering at Technical University Dresden. Her e-mail address is ella.jannasch@mailbox.tu-dresden.de.

THORSTEN SCHMIDT is a professor at Technical University Dresden and owner of the Chair of Logistics Engineering since 2008. He received his Dr.-Ing. at Dortmund University. His research interests include factory planning, factory automation and materials handling. His e-mail address is thorsten.schmidt@tu-dresden.de

Address: Institute of Material Handling and Industrial Engineering, TU Dresden, 01062 Dresden, GERMANY